OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4764

Mechanics 4

Wednesday **21 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (24 marks)

1 A spherical raindrop falls through a stationary cloud. Water condenses on the raindrop and it gains mass at a rate proportional to its surface area. At time t the radius of the raindrop is r. Initially the raindrop is at rest and $r = r_0$. The density of the water is ρ.

(i) Show that $\frac{\mathrm{d}r}{\mathrm{d}t} = k$, where k is a constant. Hence find the mass of the raindrop in terms of r_0, ρ, k and t. [6]

(ii) Assuming that air resistance is negligible, find the velocity of the raindrop in terms of r_0, k and t. [6]

2 A rigid circular hoop of radius a is fixed in a vertical plane. At the highest point of the hoop there is a small smooth pulley, P. A light inextensible string AB of length $\frac{5}{2}a$ is passed over the pulley.

A particle of mass m is attached to the string at B. PB is vertical and angle APB $= \theta$. A small smooth ring of mass m is threaded onto the hoop and attached to the string at A. This situation is shown in Fig. 2.

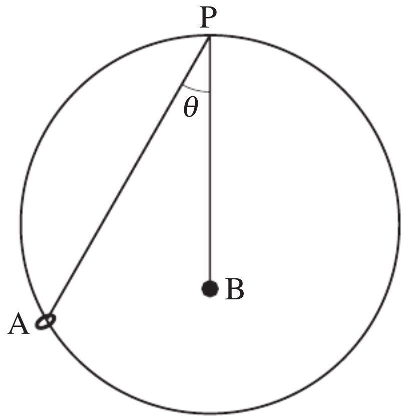

Fig. 2

(i) Show that PB $= \frac{5}{2}a - 2a\cos\theta$ and hence show that the potential energy of the system relative to P is $V = -mga(2\cos^2\theta - 2\cos\theta + \frac{5}{2})$. [4]

(ii) Hence find the positions of equilibrium and investigate their stability. [8]

Section B (48 marks)

3 An aeroplane is taking off from a runway. It starts from rest. The resultant force in the direction of motion has power, P watts, modelled by

$$P = 0.0004m(10\,000v + v^3),$$

where m kg is the mass of the aeroplane and $v\,\mathrm{m\,s^{-1}}$ is the velocity at time t seconds. The displacement of the aeroplane from its starting point is x m.

To take off successfully the aeroplane must reach a speed of $80\,\mathrm{m\,s^{-1}}$ before it has travelled 900 m.

(i) Formulate and solve a differential equation for v in terms of x. Hence show that the aeroplane takes off successfully. [8]

(ii) Formulate a differential equation for v in terms of t. Solve the differential equation to show that $v = 100\tan(0.04t)$. What feature of this result casts doubt on the validity of the model? [7]

(iii) In fact the model is only valid for $0 \leqslant t \leqslant 11$, after which the power remains constant at the value attained at $t = 11$. Will the aeroplane take off successfully? [9]

[Question 4 is printed overleaf.]

4 A flagpole AB of length $2a$ is modelled as a thin rigid rod of variable mass per unit length given by

$$\rho = \frac{M}{8a^2}(5a - x),$$

where x is the distance from A and M is the mass of the flagpole.

(i) Show that the moment of inertia of the flagpole about an axis through A and perpendicular to the flagpole is $\frac{7}{6}Ma^2$. Show also that the centre of mass of the flagpole is at a distance $\frac{11}{12}a$ from A. [8]

The flagpole is hinged to a wall at A and can rotate freely in a vertical plane. A light inextensible rope of length $2\sqrt{2}a$ is attached to the end B and the other end is attached to a point on the wall a distance $2a$ vertically above A, as shown in Fig. 4. The flagpole is initially at rest when lying vertically against the wall, and then is displaced slightly so that it falls to a horizontal position, at which point the rope becomes taut and the flagpole comes to rest.

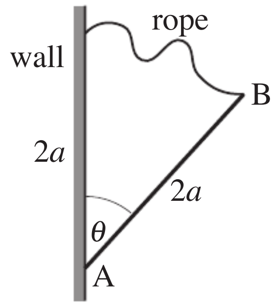

Fig. 4

(ii) Find an expression for the angular velocity of the flagpole when it has turned through an angle θ. [4]

(iii) Show that the vertical component of the impulse in the rope when it becomes taut is $\frac{1}{12}M\sqrt{77ag}$. Hence write down the horizontal component. [5]

(iv) Find the horizontal and vertical components of the impulse that the hinge exerts on the flagpole when the rope becomes taut. Hence find the angle that this impulse makes with the horizontal. [7]